ENGINEERING ECONOMY

ENGR. DARIO A. CALAQUE, JR.

ENGR. CALAQUE, D.
ENGR. DARIO CALAQUE, JR
ENGR. CALAQUE, D.
ENGR. CALAQUE, D.

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Engineering Economic Analysis

In its simplest form, an engineering economic analysis is a study of the desirability of making an investment. It is primarily inspired by the work that engineers do in performing analyses, synthesizing, and coming to a conclusion as they work on projects of all sizes. Moreover, engineering economy is at the center of making decisions. These decisions involve the fundamental elements of cash flows of money, time, and interest rates.

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INTEREST

ENGR. DARIO A. CALAQUE, JR.

ENGR. CALAQUE,D.
ENGR. DARIO CALAQUE, JR
ENGR. CALAQUE,D.
ENGR. CALAQUE,D.

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INTEREST

- SIMPLE INTEREST
  - ORDINARY SIMPLE INTEREST
  - EXACT SIMPLE INTEREST
- COMPOUND INTEREST

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SIMPLE INTEREST

$$I = Pin$$

Where: I – Interest

P – Present value, principal amount

i –Interest rate

n – Number of periods

Future Amount

$$F = P + I$$

ORDINARY SIMPLE INTEREST

$d/360$ (banker's year)

EXACT SIMPLE INTEREST

$d/365$ (ordinary year)

$d/366$ (leap year)

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LEAP YEAR

A year is a leap year if it is divisible by 4 provided it doesn't end with '00. If it ends with '00, it is only a leap year when it is divisible by 400.

LEAP YEAR MATRIX				YEAR NOT ENDING WITH '00			
1989	1993	1997	2001	2005	2009	2013	2017
1990	1994	1998	2002	2006	2010	2014	2018
1991	1995	1999	2003	2007	2011	2015	2019
1992	**1996**	**2000**	**2004**	**2008**	**2012**	**2016**	**2020**
				CENTURY YEAR			
1300	1700	2100	2500	2900	3300	3700	4100
1400	1800	2200	2600	3000	3400	3800	4200
1500	1900	2300	2700	3100	3500	3900	4300
1600	**2000**	**2400**	**2800**	**3200**	**3600**	**4000**	**4400**

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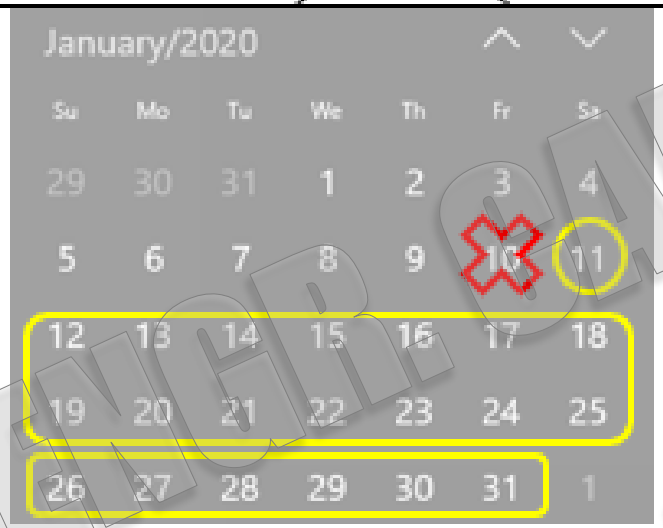
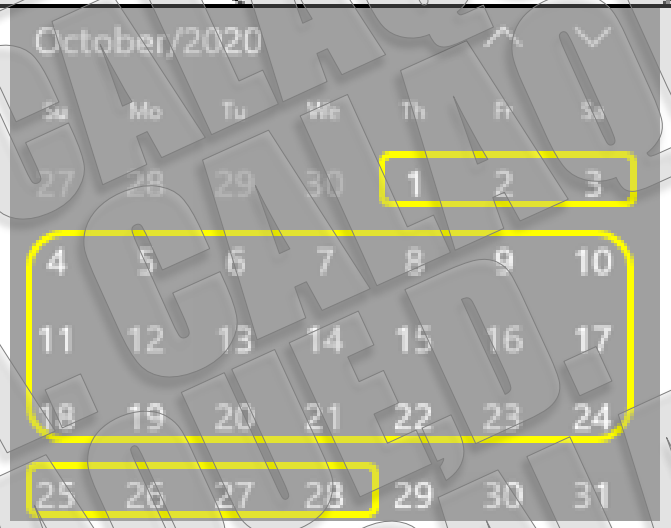
INCLUSION – EXCLUSION DATES RULE:

Consider January 10 and October 28 of the same year

Rule #01: Jan.10 (Exclude Jan.10)	Oct. 28 (Include Oct. 28)
	
No. of days = "From 11 up to 31" = 31-10 = 21days	*No. of days = "From 01 up to 28" = 28days*

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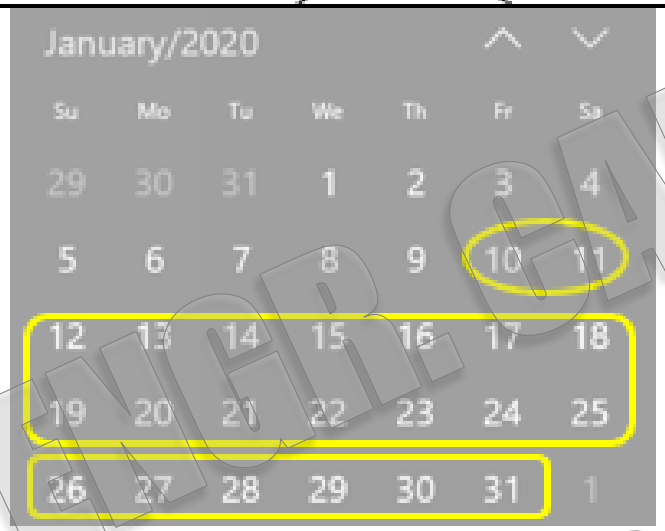
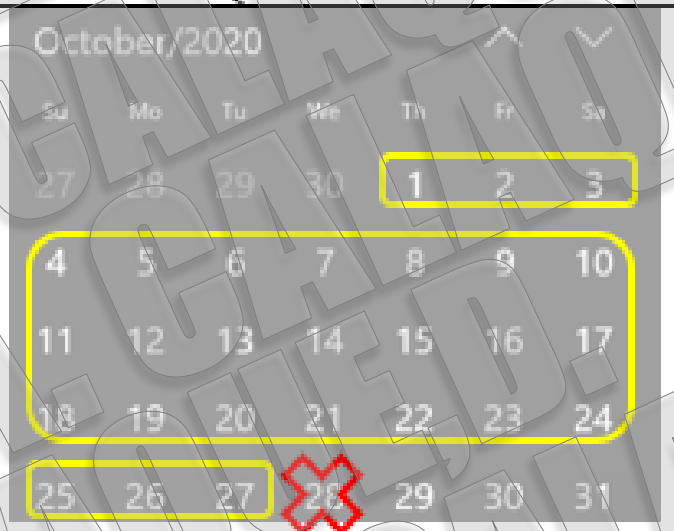
INCLUSION – EXCLUSION DATES RULE:

Consider January 10 and October 28 of the same year

Rule #02: Jan.10 (Include Jan.10)	Oct. 28 (Exclude Oct. 28)
	
No. of days = "From 10 up to 31" = 31-9=22days	*No. of days = "From 01 up to 27" = 27days*

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HAND CALENDAR

One finger represents *"31 days/month"* and the space/interface between fingers represents *"30 days/month"*. However, in the month of February, it may represents as *"28days for ordinary year"* and *"29days for a leap year"*.

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Problem:

Determine the ordinary simple interest on P10,000 for 9months and 10days if the rate of interest is 12%

SOLUTION:

Determine what type of interest (if simple or compound)

"Simple Interest"

If simple interest, determine if ordinary or exact.

"Ordinary simple interest"

If ordinary simple interest, solve for n in terms of days or in years

n= 9months and 10days

Based on Banker's Year, 1 month = 30days, 1year = 360days

n= 9(30) +10 = 280days

$$n = \frac{280}{360} = \frac{7}{9}\,yr$$

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SOLUTION:

n= 9months and 10days

Based on Banker's Year, 1 month = 30days, 1year = 360days

n= 9(30) +10 = 280days

$$n = \frac{280}{360} = \frac{7}{9}\,yr$$

I = Pin = ₱10000*0.12*(7/9) = ₱933.333

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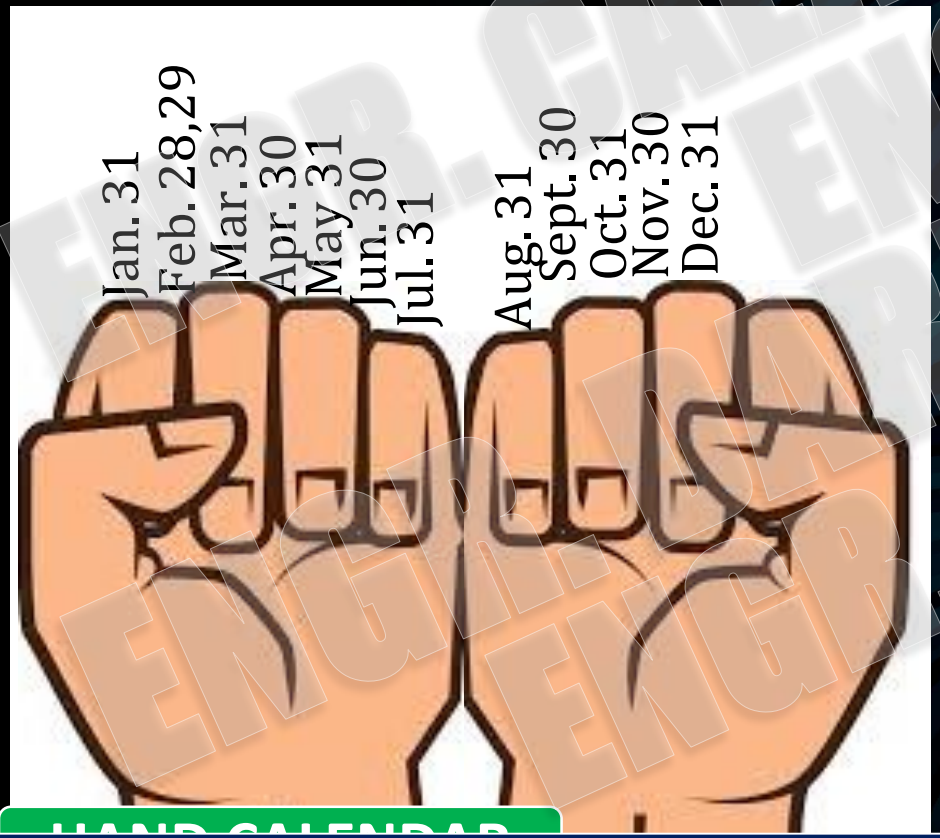
PROBLEM:

Find the ordinary and exact simple interest on ₱1,200 for the period from January 16 to November 26 , 1992 at an interest rate of 24%.

LEAP YEAR

SOLUTION:



Months	Days	
January 16 - 31		Excluding Jan.16
February		
March		
April		
May		
June		
July		
August		
September		
October		
November 26		Including Nov.26

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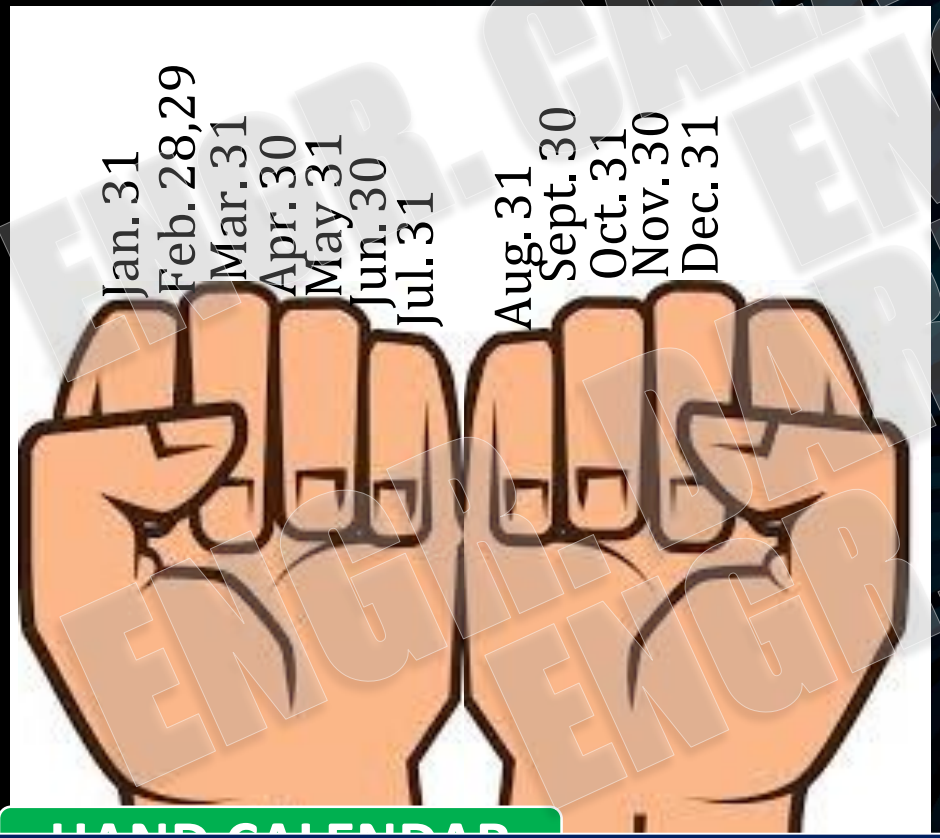
LEAP YEAR

SOLUTION:



Months	Days	
January 16 - 31		Excluding Jan.16
February		
March		
April		
May		
June		
July		
August		
September		
October		
November 26		Including Nov.26

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PROBLEM:

Find the ordinary and exact simple interest on ₱1,200 for the period from January 16 to November 26 , 1992 at an interest rate of 24%.

SOLUTION:

Months	Days
January 16 - 31	15
February	29
March	31
April	30
May	31
June	30
July	31
August	31
September	30
October	31
November 26	26
TOTAL	315

n = 315days

Ordinary Simple Interest

I = Pin

= ₱1200∗0.24∗(315/360)

= ₱252

Exact Simple Interest

I = Pin

= ₱1200∗0.24∗(315/366)

= ₱247.87

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PAST CE BOARD EXAM:

Find the exact simple interest on ₱1,000 which was deposited on January 08, 1990 and to be withdrawn on October 08, 1992 at an interest rate of 10%.

a) ₱260.85
b) ₱264.85
c) ₱274.85
d) ₱270.85

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PAST CE BOARD EXAM:

Find the exact simple interest on ₱1,000 which was deposited on January 08, 1990 and to be withdrawn on October 08, 1992 at an interest rate of 10%.

SOLUTION:

Step 1: Determine what type of interest (if simple or compound)

"Simple Interest"

Step 2: If simple interest, determine if ordinary or exact.

"Exact simple interest"

Step 3: If ordinary simple interest, solve for n in terms of year

n= Jan.08, 1990 to October 08, 1992

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PAST CE BOARD EXAM:

Find the exact simple interest on ₱1,000 which was deposited on January 08, 1990 and to be withdrawn on October 08, 1992 at an interest rate of 10%.

SOLUTION:

Step 3: If ordinary simple interest, solve for n in terms of year

n= Jan.08, 1990 to October 08, 1992

Breakdown:

1990 → Ordinary year (365days/yr.)
Jan. 08 (Excluding first date)

Jan.09, 1990 – Dec.31, 1990 = 365 – 8 = 357days = 0.978yr.

1991→ Ordinary year (365days/yr.)

Jan.01, 1991 – Dec.31, 1991 = 365days = 1yr.

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ENGR. DARIO A. CALAQUE, JR
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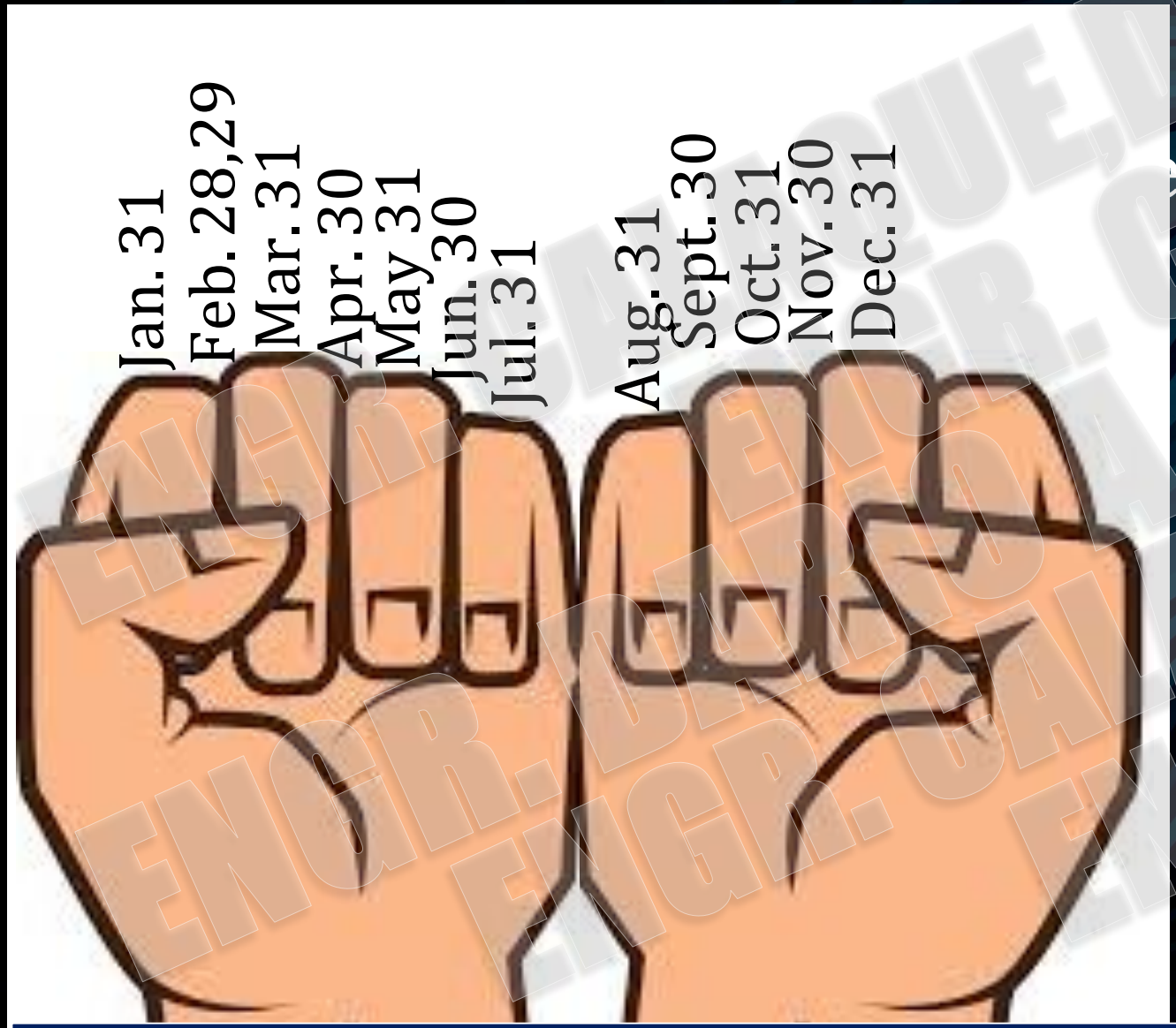

Months	Days
January 01	31
February	29
March	31
April	30
May	31
June	30
July	31
August	31
September	30
October 08	08
TOTAL	282

$n = 0.978$
$+ 1$
$+ 0.7705$
$n = 2.7485\text{yrs.}$

HAND CALENDAR

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PAST CE BOARD EXAM:

Find the exact simple interest on ₱1,000 which was deposited on January 08, 1990 and to be withdrawn on October 08, 1992 at an interest rate of 10%.

SOLUTION:

Step 1: Determine what type of interest (if simple or compound)

"Simple Interest"

Step 2: If simple interest, determine if ordinary or exact.

"Exact simple interest"

Step 3: If ordinary simple interest, solve for n in terms of year

n= Jan.08, 1990 to October 08, 1992 = 2.7485yrs.

Step 4: Solve for interest

I = Prin = ₱1000*0.10*2.7485 = ₱274.85

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PAST CE BOARD EXAM:

Find the exact simple interest on ₱1,000 which was deposited on January 08, 1990 and to be withdrawn on October 08, 1992 at an interest rate of 10%.

Find the future amount.

SOLUTION:

Step 4: Solve for interest

$$I = Pin = ₱1000*0.10*2.7485 = ₱274.85$$

Solve for the future amount

$$F = P + I = ₱1000 + ₱274.85 = ₱1274.85$$

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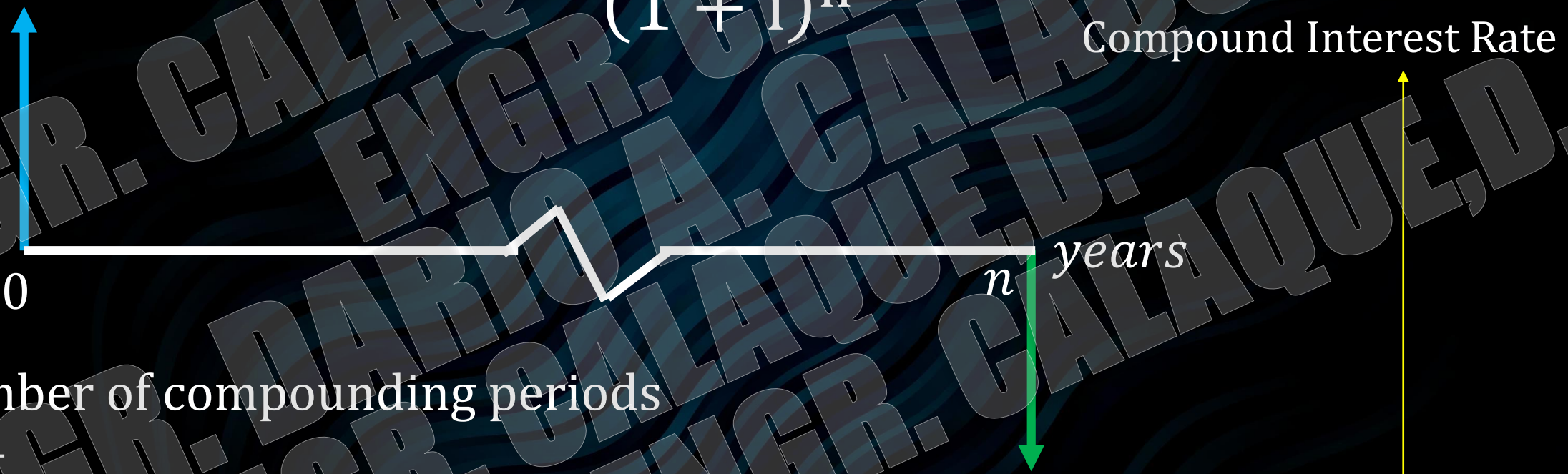
COMPOUND INTEREST

Compound interest is an interest on top of interest

Present Amount

$$P = F(1+i)^{-n} = \frac{F}{(1+i)^n}$$



$$F = P(1+i)^n$$

Future Amount

n = Number of compounding periods

$n = m\,t$

m = Number of compounding periods per year

t = Time in year(s)

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COMPOUND INTEREST RATE

NOMINAL INTEREST RATE

j % per year compounded NOT YEARLY

$\frac{j}{m}$ % per "not yearly" compounded NOT YEARLY

m = Number of compounding periods per year

EFFECTIVE INTEREST RATE

i % per year compounded YEARLY

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Common values of “m”

m	Compounding conditions	Payment Mode	Cash-Flow Basis, k
1	Annually, Yearly	Every Year	k = 12/1 = 12mos.
2	Semi-Annually, Bi- Annually	Every 6 months	k= 12/2 = 6mos.
4	Quarterly	Every 3 months	k= 12/4 = 3mos.
6	Bi-Monthly	Every 2 months	k= 12/6 = 2mos.
8	Semi-Quarterly	Every 1.5 months	k= 12/8 = 1.5mos.
12	Monthly	Every 1 month	k= 12/12 = 1.0mo.
360	Daily	Every day	k= 12/360 = 0.033mo.
∞	Continuously		k = 0.033mo. (30days/1mo.)

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CONTINUOUS COMPOUND INTEREST

$$F = Pe^{rt}$$

$$P = Fe^{-rt}$$

Where:

F = Future worth, amount after "n" periods

P = Present worth

r = nominal interest rate

t = Number of interest period, time in year(s)

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INTEREST RATE CONVERSION FORMULA

By 1Peso- 1 Year Analysis:

$$(1+i_{eff})^{1\,year} = \left(1+\frac{j}{m}\right)^{(m)(t)}$$

$$(i_{eff})^{1\,year} = \left(1+\frac{j}{m}\right)^{(m)(t)} - 1$$

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Problem:

If the sum of P12,000 is deposited in an account earning interest at the rate of 9% compounded quarterly, what will it become at the end of 8 years?

SOLUTION:

$$F = P(1+i)^n$$

$$F = 12000\left(1+\frac{0.09}{4}\right)^{4(8)}$$

$$F = ₱24,457.24$$

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Problem:

At a certain interest rate compounded quarterly, P1000 will amount to P4500 in 15 years. What is the amount at the end of 10 years?

SOLUTION:

$$F = P(1+i)^n$$

$$4500 = 1000\left(1+\frac{i}{4}\right)^{4(15)}$$

$$i = 0.10154$$

$$F = 1000\left(1+\frac{0.10154}{4}\right)^{4(10)}$$

$$F = ₱2725.7$$

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Problem:

The expansion plan of a business firm requires the purchase of a parcel of land on which to build a structure, which they will need 5 years hence. The current costs are: land, P1,000,000; building P3,500,000. Since these are not needed immediately, the firm plans to defer the purchase of the land and the construction of the building until they are needed. If the value of the land and the cost of the building are expected to appreciate at the rates of 20% and 8% per annum, respectively, what will be the total cost of the land and structure 5 years hence?

a) ₱2,488,320.000
b) ₱8,802,135.139
c) ₱5,142,648.269
d) ₱7,630,968.269

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SOLUTION:

$$F = P(1+i)^n$$

$$F_{land} = 1{,}000{,}000(1+0.2)^{(5)} = ₱2{,}488{,}320.000$$

$$F_{bldg} = 3{,}500{,}000(1+0.08)^{(5)} = ₱5{,}142{,}648.269$$

$$\text{Total Cost} = ₱7{,}630{,}968.269$$

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PAST CE BOARD EXAM:

Which of the following gives the highest annual income? Lowest annual income?

a) 9% compounded semi-annually per year

b) 9% compounded quarterly per year

c) 9% compounded bi-monthly per year

d) 9% compounded monthly per year

e) 9% compounded continuously

SOLUTION:

$$(1+i_{eff})^{1\,year} = \left(1+\frac{j}{m}\right)^{(m)(1)}$$

$$(i_{eff})^{1\,year} = \left(1+\frac{j}{m}\right)^{(m)(1)} - 1$$

Where:

i_{eff} - Effective interest rate, % per year compounded yearly, % per annum compounded annually

j – Nominal interest rate, % per year compounded not yearly, % compounded not yearly.

j/m - %per not yearly compounded not yearly.

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b) 9% compounded quarterly

c) 9% compounded bi-monthly

d) 9% compounded monthly

e) 9% compounded continuously

SOLUTION:

$$(i_{eff})^{1\,year} = \left(1 + \frac{j}{m}\right)^{(m)(1)} - 1$$

Where:

j – Nominal interest rate, % per year compounded not yearly, % compounded not yearly.

a) 9% compounded semi-annually

$$i_{eff} = \left(1 + \frac{9\%}{2}\right)^{2} - 1 = 9.2025\%$$

b) 9% compounded quarterly

$$i_{eff} = \left(1 + \frac{9\%}{4}\right)^{4} - 1 = 9.3083\%$$

c) 9% compounded bi-monthly

$$i_{eff} = \left(1 + \frac{9\%}{6}\right)^{6} - 1 = 9.3443\%$$

d) 9% compounded monthly

$$i_{eff} = \left(1 + \frac{9\%}{12}\right)^{12} - 1 = 9.3807\%$$

e) 9% compounded continuously

$$i_{eff} = e^{rt} - 1 = e^{9\%} - 1 = 9.4174\%$$

Thus, The highest annual income is 9% compounded continuously

The lowest annual income is 9% compounded semi-annually

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